18. If $\phi_1, \phi_2 \ldots \phi_n$ are n solutions of $L(y) = 0$ on an Interval I, prove that they are linearly independent there if and only if $W(\phi_1, \phi_2 \ldots \phi_n)(x) \neq 0$ for all x in I.

19. Derive second order equations with regular singular points.

20. Derive method of successive approximations.

APRIL/MAY 2024

GMA13/DMA13 — ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

Shanmuga Industries Arts & Science College T.V.Malai

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Write second order homogenous equation.

2. Define Wronskian.

3. Write non-homogeneous equation of order n.

4. State uniqueness theorem.

5. Define linear independence.

6. State existence theorem for analytic coefficients.

7. Write Lipschitz condition.

8. Write second order equation with regular singular points.

9. Write Bessel equation.

10. Write variables separated equation.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If ϕ_1, ϕ_2 are two solutions $L(y) = 0$ on an Interval I containing a point x_0, show that $w(\phi_1, \phi_2)(x) = e^{-at}(x - x_0)W(\phi_1, \phi_2)(x_0)$.

Or

(b) State and prove uniqueness theorem.

12. (a) Let $\alpha_1 \ldots \alpha_n$ be any n constants and let x_0 be any real number. Show that there exists a solution ϕ of $L(y) = 0$ on $-\infty < x < \infty$ satisfies, $\phi(x_0) = \alpha_1$, $\phi'(x_0) = \alpha_2 \ldots \phi^{n-1}(x_0)\alpha_n$.

Or

(b) Discuss about Annihilator method to find a solution of non-homogeneous equations.

13. (a) Let $\phi_1, \phi_2 \ldots \phi_n$ be n solutions of $L(y) = 0$ on a interval I_1 and let x_0 be any point in I. Show that $w(\phi_1 \ldots \phi_n) = \exp\left[-\int_{x_0}^{x} a_1(t)dt\right]$ $W(\phi_1 \ldots \phi_n)\ (x_0)$.

Or

(b) Derive the Homogenous equation with analytic coefficients.

14. (a) Derive Euler equation.

Or

(b) Derive Bessal equation.

15. (a) Derive the equations with variables separated.

Or

(b) Show that M, N be two real-valued functions with continuous first partial derivations on some rectangle

$R : |x - x_0| \le a \qquad |y - y_0| \le b$

$M(x, y) + N(x, y)y' = 0$

is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that ϕ_1, ϕ_2 be two linearly independent solution of $L(y) = 0$ on interval I. Every solutions ϕ of $L(y) = 0$ can be written uniquely as $\phi = c_1\phi_1 + c_2\phi_2$ where c_1, c_2 are constants.

17. Prove that n solution of $L(y) = 0$ given in linearly independent on any interval I.